Parametrized Quantum Circuits

Tomography and Compression

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One-shot introduction to Quantum Computing

Terms and conditions apply!

One Qubit!

- We shall deal with qubits (two-level systems)
- Can be represented as a complex linear combination of two orthogonal basis vectors

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$$

such that

$$|\alpha|^2 + |\beta|^2 = 1$$

- Measurement destroys quantum information!

Multiple qubits!

- Basis states become tensor products of the single qubit basis states.
- For two-qubit states, $|00\rangle = |0\rangle \otimes |0\rangle$, $|01\rangle$, $|10\rangle$, $|11\rangle$
- Example:

$$\frac{1}{\sqrt{2}} \left(|00\rangle + |11\rangle \right)$$

- Entanglement: states that are **not** product states

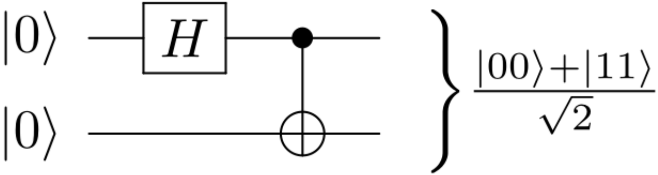
Evolution, Gates!

- Unitary matrices!
- Therefore, linear, reversible (will be used soon!)
- Examples:

$$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \qquad H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$
$$X|0\rangle = |1\rangle \qquad X|1\rangle = |0\rangle$$
$$Z|0\rangle = |0\rangle \qquad Z|1\rangle = (-1)|1\rangle$$

- Multi-qubit gates: CNOT, Flips second qubit if first qubit is 1, does nothing if not.
- You get circuits when gates act one after the other.

An example of a circuit[1]



$$|00\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |0\rangle \rightarrow \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle) \rightarrow \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

[1] Wikipedia: `https://commons.wikimedia.org/wiki/File:The_Hadamard-CNOT_transform_on_the_zero-state.png`

Parametrized Gates

- Remember title of the talk?
- **Parametrized** quantum circuits.
- X-Rotation, Z-Rotation

$$\mathrm{RX}(\theta) = \begin{bmatrix} \cos(\theta/2) & -i\sin(\theta/2) \\ -i\sin(\theta/2) & \cos(\theta/2) \end{bmatrix}$$
$$\mathrm{RZ}(\theta) = \begin{bmatrix} \exp(-i\theta/2) & 0 \\ 0 & \exp(-i\theta/2) \end{bmatrix}$$

- $\mathrm{RX}(\pi) = -i\mathrm{X}$, $\mathrm{RZ}(\pi) = -i\mathrm{Z}$

Why do people care?

- Several speed-ups ($*$) over classical algorithms are already known
- **polynomial 'time'** prime factoring (Shor's algorithm):
  - Hardness assumption of prime factoring is the basis for RSA cryptosystem
- **sub**-linear search (Grover's algorithm)
  - Unstructured search has $\Theta(n)$ complexity; Grover search has $O(\sqrt{n})$ complexity

Half-shot introduction to QML

Many flavors

- Speeding up classical machine learning algorithms using quantum algorithms for linear algebra: Harrow-Hassidim-Lloyd for solving linear equations, singular value estimation etc.
- Problems that are classically hard, eg Traveling Salesman, QAOA (quantum approximate optimization algorithm) by Farhi-Goldstone-Gutman
- Solving quantum problems, eg quantum state tomography
- Running famous quantum algorithms on near-term quantum hardware
  - Decoherence, low depth and other issues.

Parametrized quantum algorithms

Key idea: Let's use a parametrized circuit, optimize the parameters, and use the trained circuit to solve the problem.

Typically lower depth circuits for many important problems.

Computing gradients

- In many cases, the **same** quantum circuit can be used to compute the function **and** the gradient of the function
- Uses the **parameter shift rule**
- This is a generalization of...

$$f(x) = \sin(x) \Rightarrow f'(x) = \frac{1}{2}\sin\left(x + \frac{\pi}{2}\right) - \frac{1}{2}\sin\left(x - \frac{\pi}{2}\right)$$

Parametrized quantum algorithm for quantum state tomography

What is quantum state tomography?

Given several copies of a quantum state, learn (a classical description of) the state.

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$$

We want to learn α and β.

Metric is **fidelity**, in this case the **inner product** between the target and the reconstruction. We want this to be **close to** 1.

A very simple idea[2]

- Learn the parameters of a circuit that maps the state to a **known** state
- Then, use the reverse of the circuit to find what the original state was

$$U|\psi\rangle = |0\rangle \Rightarrow |\psi\rangle = U^\dagger |0\rangle$$

- Single-Shot Measurement Learning

[2] Sang Min Lee, Jinhyoung Lee, and Jeongho Bang. "Learning unknown pure quantum states". In: *Phys. Rev. A* 98 (5 Nov. 2018), p. 052302.

Single-Shot Measurement Learning

Run the circuit, measure the obtained state, then update the parameters

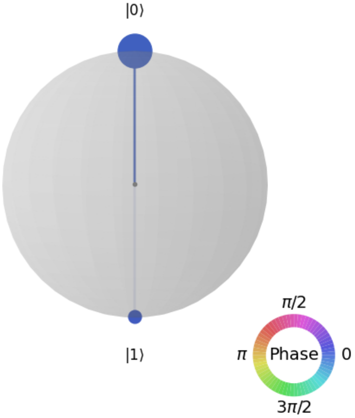
Parameter update rule is given by

$$M_S^{(n+1)}, \boldsymbol{p}^{(n+1)} = \begin{cases} M_S^{(n)} + 1, \boldsymbol{p}^{(n)} & \text{if SUCCESS} \\ 0, \boldsymbol{p}^{(n)} + \alpha (M_S^{(n)} + 1)^{-\beta} \boldsymbol{r} & \text{if FAILURE} \end{cases}$$

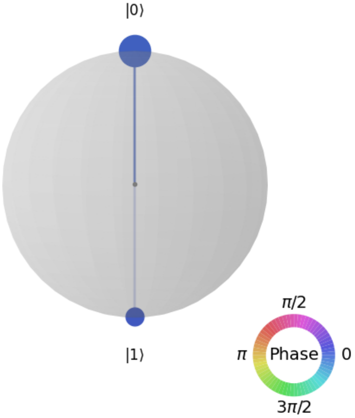
Keep doing this until $M_S^{(n)} = M_{\text{stopping}}$

Gradient free!

Simulation results (target state)



Simulation results (obtained state)



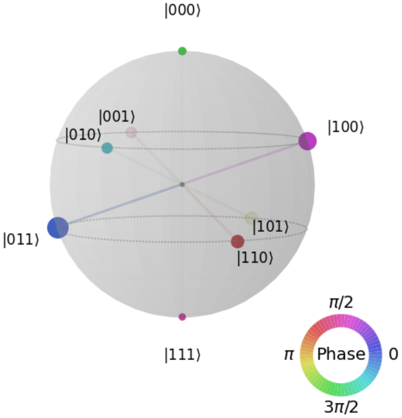
Simulation results

- Fidelity was 0.97
- **Doesn't work** for multiple qubits, unless it's product state

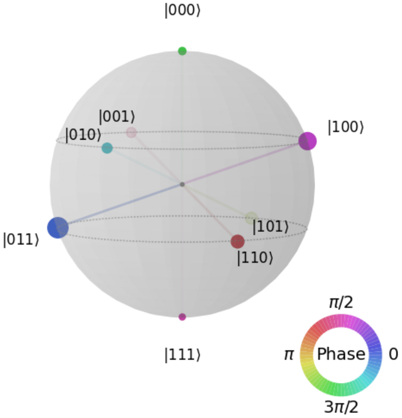
Different approach

- We can use gradient-based methods directly, using the parameter shift rule to compute gradients.
- Can use any of your favourite optimizers - gradient descent, Adagrad, Adam...

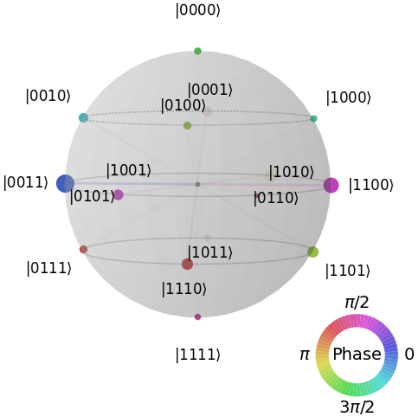
Simulation results 3 qubits (target state)



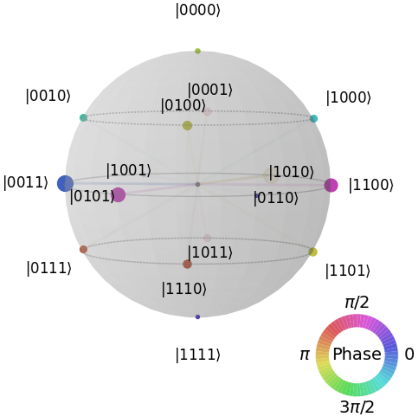
Simulation results 3 qubits (obtained state) [Fidelity - 0.999]



Simulation results 4 qubits (target state)



Simulation results 4 qubits (obtained state) [Fidelity - 0.867]



Further comments

- Fidelity starts going down for more qubits
- Most likely reason - barren plateaus
- To try: other non-gradient based optimization methods, eg particle swarm optimization

Thanks!