$X^n = (X_1, \ldots, X_n)$ n iid Bern (1/2)

Z^n :→ n iid Bern (α). $0 \leq \alpha \leq 1/2$.

$Y^n = X^n \oplus Z^n$

$\Omega = \{0,1\}$.

$\Omega_n = \{0,1\}^n$.

$b : \Omega_n \longrightarrow \Omega$.

$b^{-1}(0) = \{ x^n \in \Omega_n \mid b(x^n) = 0 \}$.

Lexicographical ordering. $<_L$

$x^k <_L \tilde{x}^k$

Notes

iff $x_j < \tilde{x}_j$ for some j and $x_i = \tilde{x}_i \; \forall \; i<j$

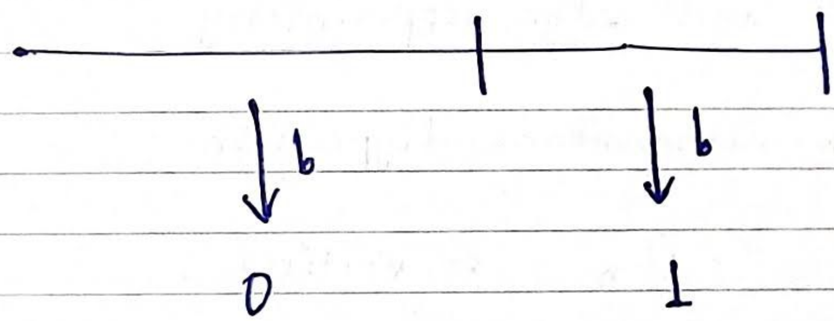
$L_k(M)$ = initial segment of size M in the lex ordering on $\{0,1\}^k$.

$L_3(4) = \{000, 001, 010, 011\}$.

(essentially converting $[M] \longrightarrow$ binary).

b is <u>lex</u> iff

$$b^{-1}(0) = L_n(|b^{-1}(0)|)$$



Notes

July

M	T	W	T	F	S	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

August

M	T	W	T	F	S	S
31					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

September

M	T	W	T	F	S	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30				

October

M	T	W	T	F	S	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

November

M	T	W	T	F	S	S
30						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29

December

M	T	W	T	F	S	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

Conjecture 2

For any given n and fixed bias $\mathbb{P}[b(x^n)=0]$ satisfying $H[\mathbb{P}[b(x^n)=0]] \geq 1-H(\alpha)$, the conditional entropy $H(b(x^n) \mid y^n)$ is minimized when b is lex.

$\rightsquigarrow$ structure

Conjecture 3 $b : \Omega_n \to \Omega$ is lex then

$$H(b(x^n) \mid y^n) \geq H(b(x^n))\, H(\alpha).$$

$\rightsquigarrow$ inequality

Conjecture 2 and edge-isoperimetry.

$Q_n \leftarrow$ n-dimensional hypercube.

$V(Q_n) = \Omega_n \leftarrow$ vertices.

$S \subseteq V(Q_n)$

Notes $\partial S \leftarrow$ number of edges that need to be deleted to disconnect S from any vertex not in S.

January							February							March							April							May							June						
M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S
			1	2	3	4							1	30	31					1			1	2	3	4	5					1	2	3	1	2	3	4	5	6	7
5	6	7	8	9	10	11	2	3	4	5	6	7	8	2	3	4	5	6	7	8	6	7	8	9	10	11	12	4	5	6	7	8	9	10	8	9	10	11	12	13	14
12	13	14	15	16	17	18	9	10	11	12	13	14	15	9	10	11	12	13	14	15	13	14	15	16	17	18	19	11	12	13	14	15	16	17	15	16	17	18	19	20	21
19	20	21	22	23	24	25	16	17	18	19	20	21	22	16	17	18	19	20	21	22	20	21	22	23	24	25	26	18	19	20	21	22	23	24	22	23	24	25	26	27	28
26	27	28	29	30	31		23	24	25	26	27	28		23	24	25	26	27	28	29	27	28	29	30				25	26	27	28	29	30	31	29	30					

Theorem 2 Harper's edge-isoperimetric inequality.

$S \subseteq V(Q_n)$ $\quad |S| = k$

$$\Rightarrow |\partial(S)| \geq |\partial(L_n(k))|$$

Proof uses compression operators

$I \subseteq [n]$ $\quad |I| = k$.

$I = \{i_1, \dots, i_k\}$ a lexicographic order

for $B \subseteq \Omega_n$ and

x^n s.t. $x_i = 0 \ \forall \ i \in I$

the I section of B at x^n is

Notes

$$B_I(x^n) = \left\{ z^k : y^n \in B, \ y_i = \begin{cases} z_j & \text{if } i = i_j \in I \\ x_i & \text{o/w} \end{cases} \right\}$$

Example

$$B = \{000, 001, 011, 101\}$$

$$B_{\{1\}}(001) = \{0, 1\}$$

$I = \{1\}$ $\quad x_i = 0$ when $i = 1$ satisfied

$$y_i = \begin{cases} z_j & \text{if } i = i_j \in I \\ x_i & \text{o/w} \end{cases}$$

01

$$B_{\{2\}}(100) = \{\phi\}.$$

$$B_{\{1,2\}}(000) = \{00\}.$$

Notes

$$B_{\{1,2\}}(001) = \{00, 01, 10\}.$$

January						
M	T	W	T	F	S	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

February						
M	T	W	T	F	S	S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	

March						
M	T	W	T	F	S	S
30	31					1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29

April						
M	T	W	T	F	S	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30			

May						
M	T	W	T	F	S	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

June						
M	T	W	T	F	S	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30					

As an algorithm what it states is the following.

$B_{\underset{\sim}{I}}(x)$ the coordinates of x that appear in I are all zero.

⇒ I is a set of indices.

⇒ match whatever is <u>not</u> in I with all the members of B.

→ for members of B that have such a match, pick up what they have in the indices in I.

I compression of B

$$= (C_I(B))_I(x^n) = L_k(|B_I(x^n)|) \quad \forall x^n$$

Notes

$C_{\mathcal{I}}$ replaces each $\mathcal{I}$ - section of B with an initial segment of the lex order.

<u>B is $\mathcal{I}$ compressed</u> if $C_{\mathcal{I}}(B) = B$.

$C_{\mathcal{I}}(B)$ is always $\mathcal{I}$ compressed.

<u>Example</u> $B = \{000, 001, 011, 101\}$

$\cancel{C_{\{1\}}(B) = L_K(B_{\mathcal{I}}(x^n))}$,

~~$B_{\mathcal{I}}(x^n)$ ⇒ Algorithmically.~~

- ~~match indices in $\mathcal{I}$ with the vectors in B.~~
- ~~wherever they match, replace with lex order.~~

$\cancel{C_{\{1,3\}}(\{000, 001, 011, 101\}) = \{000, 001, 011, 101\}}$

$I=\{1\}$

$$B_I(000) = \{0\} \rightsquigarrow \{0\}$$

$$B_I(001) = \{0,1\} \rightsquigarrow \{0,1\}$$

$$B_I(010) = \{\phi\} \rightsquigarrow \{\phi\}$$

$$B_I(011) = \{0\} \rightsquigarrow \{0\}$$

$$C_{\{1\}}(B) = \{000, 001, 011, 101\}$$

$I=\{2,3\}$

$$B_I(0\underset{\sim}{00}) = \{00, 01, 11\} \rightsquigarrow \{00, 01, 10\}$$

$$B_I(1\underset{\sim}{00}) = \{01\} \rightsquigarrow \{00\}$$

Notes

$$(C_I(B)) = \{000, 001, 010, 100\}$$

July

M	T	W	T	F	S	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

August

M	T	W	T	F	S	S
31					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

September

M	T	W	T	F	S	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30				

October

M	T	W	T	F	S	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

November

M	T	W	T	F	S	S
30						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29

December

M	T	W	T	F	S	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

$$C_{\{1,3\}}(\{000, 001, 011, 101\})$$

$$= \{000, 001, 100, 010\}$$

Algorithmically, collect all members of B that are the same at points outside I, replace points in I by lex orders.

$$A = \{000, 010, 011, 111\}$$

$$C_{\{2\}}(A) = \{000, 010, 001, 101\}$$

$$C_{\{3\}}(A) = \{000, 001, 010, 100\}$$

Notes

January

M	T	W	T	F	S	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

February

M	T	W	T	F	S	S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	

March

M	T	W	T	F	S	S
30	31					1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29

April

M	T	W	T	F	S	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30			

May

M	T	W	T	F	S	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

June

M	T	W	T	F	S	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30					

Observations

- $|C_{\mathcal{I}}(B)| = |B|$
- B is $\mathcal{I}$ compressed
- $\Rightarrow$ B is $\mathcal{I}$ compressed $\forall\, J \subset \mathcal{I}$.

Theorem 3

$b : \Omega_n \to \Omega$

$\mathcal{I} \subseteq \{1, \dots, n\}$ satisfy $|\mathcal{I}| = 2$

If $\hat{b} : \Omega_n \to \Omega$

s.t. $\hat{b}^{-1}(0) = C_{\mathcal{I}}(b^{-1}(0))$

then

$$I(\hat{b}(X^n); Y^n) \geq I(b(X^n); Y^n)$$

Notes

If C_I changes an element of $b^{-1}(0)$, it moves it lower in lexicographical order.

If we apply thm for different $|I| = 2$

$\rightarrow$ terminates at $\hat{b}$ which is I-compressed $\forall I$ with $|I| \leq 2$.

<u>Corollary</u> $S_n = \{ b : \Omega_n \rightarrow \Omega$ for which $b^{-1}(0)$ is I compressed $\forall I$ s.t $I \leq 2\}$

In maximizing $I(b(X^n); Y)$ we can stick to $b \in S_n$.

Allows enumeration of functions in S_n.

Notes and evaluating $I(b(X^n); Y^n)$ $b \in S_n$.

Gives intuition for ~~Thm~~ conjecture (2).

because applying $\mathcal{I}$-compression to $b^{-1}(0)$ yields a function $\hat{b}$ that is

i) closer to being lex.

ii) given $|\mathcal{I}| \leq 2$ satisfies

$$H(\hat{b}(x^n) \mid y^n) \leq H(b(x^n) \mid y^n)$$

$|\mathcal{I}| = n$ proves the conjecture.

But,

counterexamples where compression increases $H(b(x^n) \mid y^n)$ for $|\mathcal{I}| > 2$ but reduces $H(b(x^n) \mid y^n)$ for $|\mathcal{I}| = n$.

Notes

July

M	T	W	T	F	S	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

August

M	T	W	T	F	S	S
31						
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

September

M	T	W	T	F	S	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30				

October

M	T	W	T	F	S	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

November

M	T	W	T	F	S	S
30						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29

December

M	T	W	T	F	S	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

Rem 1

Threshold function and local maximum in terms of mutual information provided about Y^n.

Rem 2 $b : \Omega_n \to \Omega$ is monotone if

$b(x^n) \leq b(\tilde{x}^n)$ when $x_i \leq \tilde{x}_i$ for

$1 \leq i \leq n$.

monotone boolean functions are $\mathcal{I}$-compressed $\forall$

$\mathcal{I}$ with $|\mathcal{I}| = 1$.

Cor 1 $\Rightarrow$ $I(b(X^n) ; Y^n)$ is maximized by a monotone function.

Th 3 $\Rightarrow$ we may consider only regular functions.

Notes

Proof of theorem 1

$X^n \sim$ iid Bern(1/2).

$Y^n \sim X^n + Z^n \sim BSC(\alpha)$.

$b : \{0,1\}^n \to \{0,1\}$

with $\mathbb{P}[b(X^n) = 0] = 1/2$

we have $\sum_{i=1}^{n} I(b(X^n); Y_i) \leq 1 - H(\alpha)$.

lemma $\rho^2 \leq 1$

$\rho \in [0,1]$

an optimal solution for the non-convex program

Notes

July						
M	T	W	T	F	S	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

August						
M	T	W	T	F	S	S
31					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

September						
M	T	W	T	F	S	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30				

October						
M	T	W	T	F	S	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

November						
M	T	W	T	F	S	S
30						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29

December						
M	T	W	T	F	S	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

$$\min \sum_{i=1}^{n} H\left(\frac{1+\rho x_i}{2}\right)$$

$$\text{s.t.} \quad \sum_{i=1}^{n} x_i^2 \le r^2$$

is given by $x_i = p$ and $x_i = 0$ for $i = 2, \dots, n$

<u>Proof</u> Taylor expand H around $1/2$

$$\sum_{i=1}^{n} H\left(1+\frac{\rho x_i}{2}\right)$$

$$= n - \log e \sum_{k=1}^{\infty} \sum_{i=1}^{n} \frac{(\rho x_i)^{2k}}{(2k-1)\, 2k}$$

for fixed k, the sum

$$\sum_{i=1}^{n} x_i^{2k} \quad \text{is convex in} \quad x_i^2$$

Notes

January						
M	T	W	T	F	S	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

February						
M	T	W	T	F	S	S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	

March						
M	T	W	T	F	S	S
30	31					1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29

April						
M	T	W	T	F	S	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30			

May						
M	T	W	T	F	S	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

June						
M	T	W	T	F	S	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30					

and maximized subject to $\sum_{i=1}^{n} x_i^2 < r^2$

when $x_1 = r$ and $x_i = 0$ for $i = 2, \dots, n$.

Proof of Theorem 1

X^n, Y^n take values in $\{-1, 1\}^n$

$$b : \{-1, 1\}^n \to \{-1, 1\}$$

Any boolean function b can be written as

$$b(x^n) = \sum_{S \subseteq [n]} \hat{b}(S) \prod_S (x^n)$$

$$\hat{b}(S) = \mathbb{E}\, b(X^n) \prod_S (X^n)$$

Notes

$$S = \phi \Rightarrow \prod_S (x^n) = 1.$$

① For all x^n, $\prod_S (x^n)^2 = 1$.

$$\mathbb{E} \prod_S (x^n)^2 = 1.$$

② If $S \neq T$ and $S \neq \phi$ $T \neq \phi$ then $\prod_S (X^n)$ and $\prod_T (X^n)$ are independent and uniformly distributed on $\{-1, 1\}$

$$\mathbb{E} \prod_S (X^n) \prod_S (X^n) = 0.$$

If $\phi = S \neq T$ then

$$\mathbb{E} \prod_S (X^n) \prod_T (X^n).$$

$$= \mathbb{E} \prod_T (X^n) = 0.$$

Notes

January						
M	T	W	T	F	S	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

February						
M	T	W	T	F	S	S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	

March						
M	T	W	T	F	S	S
30	31					1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29

April						
M	T	W	T	F	S	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30			

May						
M	T	W	T	F	S	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

June						
M	T	W	T	F	S	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30					

③ Parseval's theorem.

$$1 = \mathbb{E}\, b(x^n)^2$$

$$= \sum_{S \subseteq [n]} \hat{b}(S)^2.$$

$$\mathbb{E}\left[b(x^n)^2 \right]$$

$$= \mathbb{E}\left[\left(\sum_S \hat{b}(S) \prod_S (x^n) \right) \left(\sum_T \hat{b}(T) \prod_T (x^n) \right) \right]$$

$$= \mathbb{E} \sum_S \sum_T \hat{b}(S)\, \hat{b}(T)\, \mathbb{E}\left[\prod_S (x^n) \prod_T (x^n) \right]$$

$$= \sum_S \hat{b}(S)^2$$

Notes ④ $\hat{b}(\phi) = \mathbb{E}\left[b(x^n) \right] = 0$

since $b(x^n)$ is equiprobable.

$$\mathbb{P}\left[b(X^n) = 1 \mid Y_i = y_i \right]$$ easy to see on expanding this

$$= \frac{1 + \mathbb{E}\left[b(X^n) \mid Y_i = y_i \right]}{2}$$

$$= \frac{1 + \mathbb{E}\left[\sum_S \hat{b}(S)\, \pi_S(X^n) \mid Y_i = y_i \right]}{2}$$

$$= \frac{1 + \sum_S \hat{b}(S)\, \mathbb{E}\left[\pi_S(X^n) \mid Y_i = y_i \right]}{2}$$

$\boxed{\text{fix rest and flip one value}}$

$= 0$ unless $S = \phi$ or $S = \{i\}$ since $\{X_j\}_{j \in S \setminus \{i\}}$ are independent of Y_i

Notes

$$= \frac{1 + \hat{b}(\phi) + \rho y_i \hat{b}(\{i\})}{2} \checkmark$$

January

M	T	W	T	F	S	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

February

M	T	W	T	F	S	S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	

March

M	T	W	T	F	S	S
30	31					1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29

April

M	T	W	T	F	S	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30			

May

M	T	W	T	F	S	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

June

M	T	W	T	F	S	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30					

$$= \frac{1 + \rho y_i \hat{b}(\{i\})}{2}$$

$$\rho \triangleq \mathbb{E}[X_i \mid Y_i = 1] = 1 - 2\alpha$$

$$\sum_{i=1}^{n} I(b(X^n); Y_i)$$

$$= nH(b(X^n)) - \sum_{i=1}^{n} H(b(X^n) \mid Y_i)$$

equiprobable

$$= n - \underbrace{\sum_{i=1}^{n} H(b(X^n) \mid Y_i)}_{}$$

minimize subject to $\mathbb{P}[(b(X^n) = 1) = 1/2]$

$$\sum_{i=1}^{n} H(b(X^n) \mid Y_i)$$

$$= \cancel{\sum_{i=1}^{n} \frac{1}{2} [H(b(X^n) \mid Y_i = 1)}$$

Notes

July							August							September							October							November							December						
M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S
		1	2	3	4	5	31					1	2		1	2	3	4	5	6				1	2	3	4	30						1		1	2	3	4	5	6
6	7	8	9	10	11	12	3	4	5	6	7	8	9	7	8	9	10	11	12	13	5	6	7	8	9	10	11	2	3	4	5	6	7	8	7	8	9	10	11	12	13
13	14	15	16	17	18	19	10	11	12	13	14	15	16	14	15	16	17	18	19	20	12	13	14	15	16	17	18	9	10	11	12	13	14	15	14	15	16	17	18	19	20
20	21	22	23	24	25	26	17	18	19	20	21	22	23	21	22	23	24	25	26	27	19	20	21	22	23	24	25	16	17	18	19	20	21	22	21	22	23	24	25	26	27
27	28	29	30	31			24	25	26	27	28	29	30	28	29	30					26	27	28	29	30	31		23	24	25	26	27	28	29	28	29	30	31			