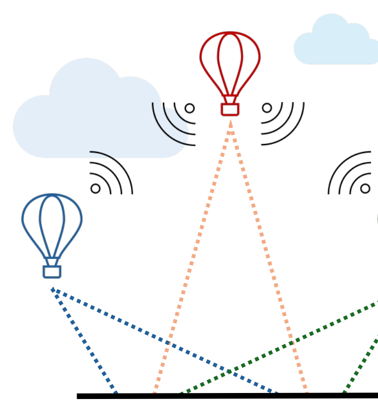
Background

My PhD research in information theory is on **shared information** (SI), that generalizes Shannon's mutual information to multiple random variables. When multiple terminals have access to correlated data and can broadcast data to each other, shared information characterizes:

- the maximum rate of shared secret key that the terminals can generate independent of the communication, and
- the minimum rate of communication for omniscience, i.e., for each terminal to reconstruct the information they collectively see.



Problem

The shared information of random variables $X_1, X_2, \ldots, X_m$ is:

$$\mathrm{SI}(X_1, \ldots, X_m) = \min_{\pi} \frac{1}{|\pi| - 1} \left[\sum_{l=1}^{|\pi|} \mathrm{H}(X_{\pi_l}) - \mathrm{H}(X_1, \ldots, X_m) \right]$$

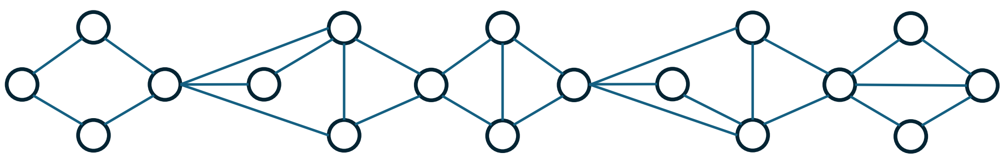
All nontrivial partitions

partition size

atom of the partition

An example system: Weather balloons (terminals) observe correlated data (overlapping parts of the ground) and communicate among themselves to create a shared map. The optimal rate of communication is the rate of communication for omniscience.

There is an efficient algorithm to compute this for a known joint distribution, but for unknown joint distributions the optimization is **intractable**. Bell(n) = number of partitions grows exponentially!



Requires checking Bell (18) = 682076806159 partitions!

Can we make the optimization more efficient in graphical models, and estimate shared information from data more efficiently?

Known: in **trees**, optimal partition is obtained by cutting the edge with the **least mutual information**

Computation

Randomly generated joint distributions with a given graph structure in Python and computed the optimal partitions

Reimplemented 3 times:
- Vectorization using Numpy
- Multithreading
- Julia

200× faster Python code
- more joint distributions
- larger graphs
- faster iteration in testing hypotheses

Results

Simulated optimal partitions always consisted of **connected atoms** (in the graph) and **connected complements of atoms**

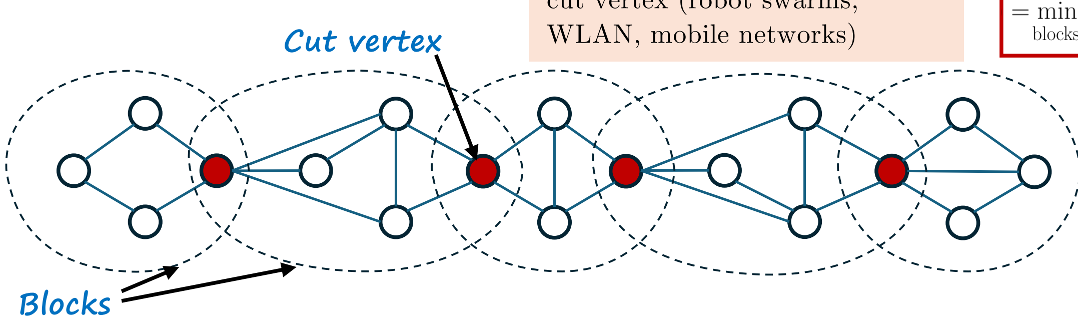
A new and more elegant proof for trees

Structural result for a class of nontree graphical models, called cliqueylons

Formula for any graph with a cut vertex (robot swarms, WLAN, mobile networks)

My 200x faster simulation tool was the key, enabling rapid testing of hypotheses and directly led to the theoretical breakthroughs.

$$\mathrm{SI}(\text{graphical model}) = \min_{\text{blocks}} \mathrm{SI}(\text{model blocks})$$



Requires checking 149 = Bell(4)×3 + Bell(5)×2 partitions, down from 682 trillion!